Modal Confidence Factor in Vibration Testing

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The modal confidence factor (MCF) is a number calculated for every identified mode for a structure under test. The MCF varies from 0.00 for a distorted nonlinear, or noise mode to 100.0 for a pure structural mode. The theory of the MCF is based on the correlation that exists between the modal deflection at a certain station and the modal deflection at the same station delayed in time. The theory and application of the MCF are illustrated by two experiments. The first experiment deals with simulated responses from a two-degree-of-freedom system with 20%, 40%, and 100% noise added. The second experiment was run on a generalized payload model. The free decay response from the payload model contained 22% noise.

Introduction

IN modal vibration testing of complex structures, there exists some level of uncertainty as to the identified modal parameters in spite of the method or technique used to extract these modes. This uncertainty arises because of nonlinearity of the structure under test, high coupling between closely spaced modes, and/or high levels of noise in the data used.

The theory and applications of a "time domain" modal test technique were presented in Refs. 1-3. The method uses free decay or random responses[4] from a structure under test to identify its modal characteristics, namely, natural frequencies, damping factors, and mode shapes. The method was proven to be accurate, economical, and insensitive to high levels of noise in the data. Furthermore, the method can identify multimodal (highly coupled) systems and modes that have very small contribution in the responses.

In Ref. 3, a method is presented to decrease the effects of high levels of noise in the data and thus improve the accuracy of identified parameters. This is done by using an oversized mathematical model. If the responses to be used are thought to have m number of modes, a mathematical model to identify $(m+n)$ modes is used. This gives n exits for noise, and the m modes can be identified more accurately.

In this paper, the concept of modal confidence factor (MCF) is developed. MCF is a factor computed for every identified mode, and this factor should be unity for any linear structural mode. Using this MCF, the clean structural modes can be separated from noise modes without leaving much chance for personal judgement.

The theory of the MCF is based on using the response of a station on the structure under test, $x(t)$, and the same response delayed $\Delta\tau$, $x(t+\Delta\tau)$ (a transformed station), in the identification program. For every identified mode, the MCF is calculated as a function of the modal deflections at the original station and the transformed station, the frequency and damping for that mode, and the delay time $\Delta\tau$.

Simulated and experimental results are reported in support of the MCF theory developed in this paper. It is important to note that, although the MCF is developed to be used in conjunction with the "time domain" identification technique, the same concept can be adapted for use with other vibration identification techniques.

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Background

A. Time Domain Identification Technique

This technique is described fully in Refs. 2 and 3. It uses the free responses (free decay) of a structure under test to identify its vibration parameters, namely, frequencies, damping factors, and modal vectors in complex form. From the measured free responses of n stations on a structure under test, assuming that the responses contain n modes, a matrix A is formed such that

$$A = \begin{bmatrix} Y \\ Z \end{bmatrix} \begin{bmatrix} X \\ Y \end{bmatrix}^{-1} \tag{1}$$

where

$$x_{ij} = x_i(t_j)$$

$$y_{ij} = x_i(t_j + \Delta t)$$

$$z_{ij} = x_i(t_j + 2\Delta t)$$

$$i = 1,n \text{ and } j = 1,2n$$

The eigenvectors of the A matrix are the modal vectors, and the eigenvalues α_i are related to the characteristic roots λ_i of the system through the equation

$$\alpha_i = \exp(\lambda_i \Delta t) \tag{2}$$

This technique was originally developed such that, for unique identification, the order of the matrix A should be equal to twice the number of modes excited in the measured response x.

In case of noisy data, two methods were presented in Ref. 3 to reduce the effect of noise on identified parameters. One of these methods was to use a mathematical model of order higher than the number of modes excited in the responses used for identification. The difference between the order of the mathematical model and the number of modes in the responses is merely an exit for the noise in the data. Using an oversized mathematical model was proven to be very effective in reducing the effect of high levels of noise on identified parameters.

B. Transformed Stations

The time domain identification technique was developed to use the minimum amount of measuring channels. Any structure, however complex, can be identified using only two measuring stations at a time, with one station kept as a reference station all of the time.[2] Because of this, a situation will arise when the number of modes excited in the measured responses is greater than the number of measuring stations. In such a case, it is not necessary to physically add measurement equipment and repeat the experiment; it is possible to use "assumed stations," for which the responses are generated using the response of the original, or real, stations. Indeed, if $x(t)$ is the response of the p real stations on a structure and m is the number of modes in the response, the response at p assumed stations can take the form of $x(t+\Delta\tau)$. This can be shown by writing the response at time t_j as

$$x(t_j) = \sum_{i=1}^{2m} R_i \psi_i \exp(\lambda_i t_j) \tag{3}$$

and, at time $(t_j+\Delta\tau)$,

$$x(t_j+\Delta\tau) = \hat{x}(t_j) = \sum_{i=1}^{2m} R_i \exp(\lambda_i \Delta\tau) \psi_i \exp(\lambda_i t_j) \tag{4}$$

where R_i is a constant associated with modal vector ψ_i, and λ_i is the ith characteristic root. The two responses of Eqs. (3) and (4) can be written as

$$\begin{bmatrix} x(t_j) \\ \hat{x}(t_j) \end{bmatrix} = \sum_{i=1}^{2m} R_i \begin{bmatrix} \psi_i \\ \hat{\psi}_i \end{bmatrix} \exp(\lambda_i t_j) \tag{5}$$

where

$$\hat{\psi}_i = \exp(\lambda_i \Delta t) \psi_i \tag{6}$$

Equation (5) may be considered to be the response vector for a system with $2p$ stations and m modes. This procedure can be repeated to increase the apparent number of stations to $3p$, $4p$,..., etc.

Theory of the Modal Confidence Factor

In typical modal testing of a structure, if $x(t)$ is the measured free responses from p station on the structure under test, new responses $\hat{x}(t) = x(t+\Delta\tau)$, $\hat{\hat{x}}(t) = x(t+2\Delta\tau)$,..., etc., will be generated to increase the apparent number of stations to be used in the identification program. To calculate the MCF for a specific mode, one of the structure's stations will be arbitrarily specified, together with the same station delayed in time $\Delta\tau$, say. If λ is the identified characteristic root for the mode, Q is the identified modal deflection at the chosen structure's station, and $\hat{Q}$ is the identified modal deflection at the same station delayed $\Delta\tau$, then these quantities can be used to calculate the MCF for the mode under consideration. From the theory of transformed stations discussed in the preceding section, the modal deflection expected at the transformed station should be

$$\hat{Q}_{\text{expected}} = Q \exp(\lambda\Delta\tau)$$

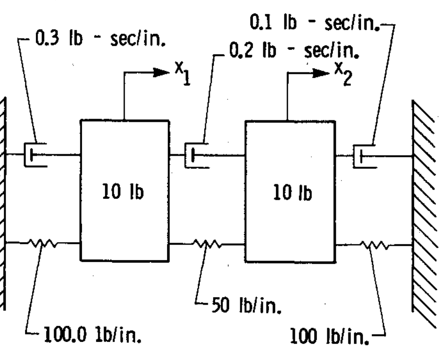

Fig. 1 Two-degree-of-freedom system.

If the identified mode is a clean linear structural mode, $\hat{Q}$ should be equal to $\hat{Q}_{\text{expected}}$. Generally,

$$\hat{Q}_{\text{expected}} = \hat{Q} \times (\text{MCF})$$

from which the equation for the MCF will be

$$(\text{MCF}) = \left|\frac{\hat{Q}_{\text{expected}}}{\hat{Q}}\right| \times 100 \qquad \hat{Q} > \hat{Q}_{\text{expected}}$$

$$= \left|\frac{\hat{Q}}{\hat{Q}_{\text{expected}}}\right| \times 100 \qquad \hat{Q}_{\text{expected}} > \hat{Q}$$

Experimental Results

A. Simulated Experiment on a Two-Degree-of-Freedom System

Free decay responses from the system shown in Fig. 1 were simulated on the CDC-6600 computer. Generated random numbers were added to the responses as noise. Three different noise levels were added such that the root-mean-square values of noise/signal ratio were 20%, 40%, and 100%. Figure 2 shows the free decay responses $x_1(t)$ and $x_2(t)$ without noise and with three different levels of noise added.

The responses were used to identify the modal parameters of the system using the time domain technique described in Ref. 3. Different orders for the mathematical model were assumed. Mathematical models of 2, 4, 6, 8, and 10 degrees of freedom were used. The MCF for different cases were calculated. Results are summarized in Tables 1 and 2. In Table 1, the MCF for the first two modes, the real system's modes, is quite high. For the rest of the modes MCF is low, indicating nonstructural modes or noise modes. The identified

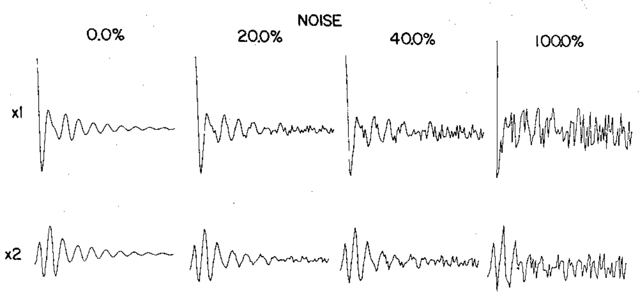

Fig. 2 Two-degree-of-freedom response.

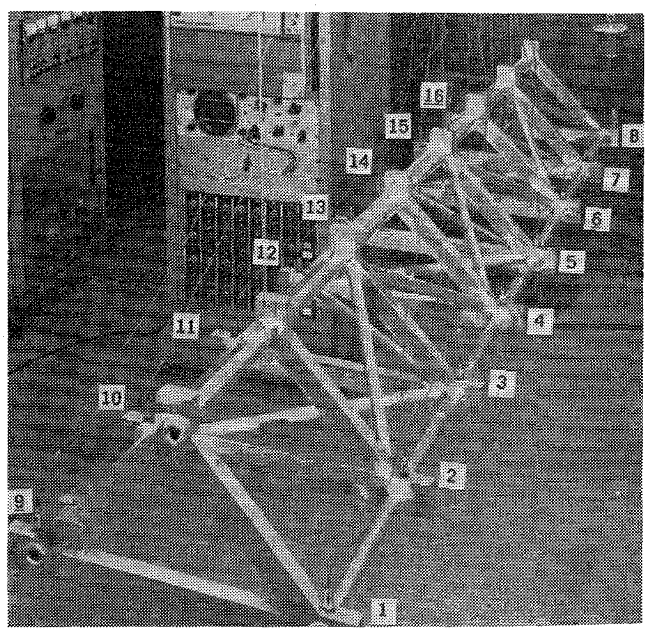

Fig. 3 Payload model.

Table 1 MCF for the two-degree-of-freedom system

No.	No. of modes								
	20% noise				40% noise				100% noise
of modes	4	6	8	10	4	6	8	10	10
1	90.0	98.3	98.7	99.6	81.7	93.7	96.4	97.7	89.9
2	96.7	96.4	96.3	92.3	98.0	93.6	97.1	89.6	70.4
3	32.0	24.2	26.0	20.4	30.8	24.5	25.5	21.5	19.6
4	8.2	15.0	6.0	45.2	5.6	16.5	5.1	44.0	22.0
5	...	10.6	10.4	17.8	...	14.6	10.3	16.2	38.9
6	...	0.2	8.6	17.5	...	4.0	9.2	16.3	8.2
7	...	...	4.6	7.1	...	...	4.4	6.6	7.1
8	...	...	3.6	8.7	...	...	3.3	8.8	12.0
9	...	...	...	8.9	...	...	...	7.4	1.6
10	...	...	...	0.1	...	...	...	0.1	3.0

Table 2 Identified frequencies and damping factor for the two-degree-of-freedom system

		Number of modes assumed								
	Theoretical	20% noise				40% noise				100% noise
Parameter	2	4	6	8	10	4	6	8	10	10
f_1	9.86	9.87	9.85	9.86	9.86	9.88	9.85	9.87	9.86	9.87
% error	...	0.1	0.1	0.0	0.0	0.2	0.1	0.1	0.0	1.1
λ_1	6.2	5.9	6.4	6.3	6.4	6.0	6.8	7.0	6.8	9.7
% error	...	4.8	3.0	2.0	3.0	3.0	10.0	13.0	10.0	58.0
f_2	13.79	13.68	14.09	13.78	13.79	13.60	14.36	13.82	13.9	14.38
% error	...	9.8	2.0	0.1	0.0	1.4	4.1	0.2	0.8	4.3
λ_2	13.2	14.4	13.2	13.4	13.4	16.90	12.8	14.3	14.2	16.6
% error	...	9.8	0.0	1.5	1.5	28.0	3.0	8.3	7.6	25.8

Table 3 MCF for payload model

	20 Modes		30 modes			40 modes			
	Good	Bad	Good	Bad	Bad	Good	Bad	Bad	Bad
1	93.5	0.0	97.5	0.0	0.0	97.7	0.0	0.0	11.6
2	98.3	0.0	98.2	0.0	0.0	98.2	0.0	0.1	9.6
3	97.8	0.0	99.2	0.0	0.2	99.8	0.0	0.1	18.6
4	98.2	0.0	98.6	0.0	2.5	98.5	0.0	0.5	12.3
5	94.8	0.0	97.0	0.0	7.5	97.8	0.0	0.2	25.3
6	94.0	0.0	96.1	0.0	3.9	96.5	0.0	0.4	72.9
7	81.0	0.2	88.9	0.0	19.5	87.3	0.0	1.3	79.2
8	96.2	1.6	99.6	0.0	63.4	99.0	0.0	0.6	...
9	75.6	31.1	89.0	0.0	...	92.0	0.0	1.6	...
10	92.9	...	93.0	0.0	...	93.1	0.0	3:4	...
11	97.9	...	98.0	0.0	...	97.9	0.0	3.3	...

frequencies and damping factors are listed in Table 2, together with the percentage error. The errors in the identified parameters are much lower that the percentage of noise in the responses used for identification, even in the case of 100% noise.

B. Generalized Payload Model

The payload model is shown in Fig. 3. Sixteen accelerometers were fixed to the eight bulkheads, eight accelerometers on each side (Fig. 3). Two data groups were used. Data group 1 had accelerometers 1 to 8. Data group 2 had accelerometers 9 to 16 and accelerometers 8 as a common accelerometer for the two data groups. A random input was applied at station 8. The input was cut off, and free responses from data group 1 were recorded on a tape recorder. The procedure was repeated for data group 2. A two-way switch was used to cut off the random input and at the same time generate a dc signal of about 1 V. The start of the dc signal, recorded on a separate channel of the tape recorder, was used to determine the start of the free response.

The free responses were filtered to eliminate frequency components higher than 350 Hz and then digitized at a sampling rate of 2000 samples per second. Only 500 points for each channel were stored to be used as data for the identification program. This corresponds to a record length of 0.25 second.

The noise/signal ratio for the resulting data was estimated at about 22%. This estimate was based on comparing two responses from station 8 which were recorded simultaneously on two channels of the tape recorder. The root mean squares of the two records, rms and RMS, were calculated, and the noise/signal ratio was estimated using the following formula:

$$N/S = \sqrt{\frac{(\mathrm{RMS} - \mathrm{rms})^2}{\mathrm{RMS} \times \mathrm{rms}}}$$

Higher-order response vectors were generated by delaying the responses of the original 17 stations 0.005 and 0.01 seconds; thus the apparent number of stations is 51. These responses were used as data for the time domain identification program. Three cases were studied, for each of which the size of the mathematical model was different. Mathematical models of 20, 30, and 40 degrees of freedom were used. Modal characteristics were identified, together with the MCF for each mode. Table 3 lists the MCF for the three cases. In all

Table 4 Identified frequencies for the payload model

Mode no.	Time domain 20 DOF[a]	30 DOF	40 DOF	Frequency sweep	NASTRAN	FFT
1	74.2	74.2	74.1	74.6	73.4	74.1
2	78.7	78.7	78.7	79.7	80.1	78.8
3	119.9	119.8	119.8	120.7	117.3	119.6
4	156.6	156.6	156.6	158.5	158.9	156.5
5	162.1	161.9	161.9	163.1	159.9	161.6
6	216.0	216.4	216.4	219.2	218.6	216.5
7	245.4	245.4	245.2	246.7	244.6	245.0
8	258.9	259.2	259.3	?	253.1	259.0
9	260.1	260.6	261.0	263.7	...	261.0
10	280.9	280.9	280.9	283.7	283.0	281.0
11	325.3	325.3	325.3	325.0	...	325.0

[a] DOF = degree-of-freedom.

of the cases, the MCF was consistently high for the first 11 modes and very low for the rest of the modes, indicating 11 modes excited in the responses used. Table 4 lists the identified frequencies from the time domain technique, together with frequencies identified using frequency sweep, NASTRAN, and FFT.

Conclusions

The modal confidence factor (MCF) is a very powerful tool in the identification of modal characteristics of structures. It is especially useful when the data used have high levels of noise. The MCF can differentiate between a real structural model and a noise mode.

In the two experiments reported in this paper, the MCF ranged from 0.0 for a noise mode to over 99.0 for a good or pure structural mode. The theory and computation procedure are simple, and calculation of the MCF is an integral part of the identification program.

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